STEADY-STATE HEAT TRANSFER THROUGH A WALL WITH LONGITUDINAL RECTANGULAR FINS AND VARIABLE THERMAL CONDUCTIVITY

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A method is examined of solving steady-state problems of heat transfer through a surface with longitudinal rectangular fins in the case when the thermal conductivity depends on temperature.

The widespread occurrence in modern technology of processes accompanied by liberation of large amounts of heat requires the careful study of methods of heat removal. The importance of using cooling fins or protruding surfaces to intensify heat transfer has long been known. The usual solution to steady-state heat transfer problems is derived on the assumption that the thermal conductivity does not depend on temperature. For comparatively small temperature differences between the faces of the fin this assumption leads to small errors in the solution, but for large temperature differences, 600°–1000° K, the error may be substantial [3].

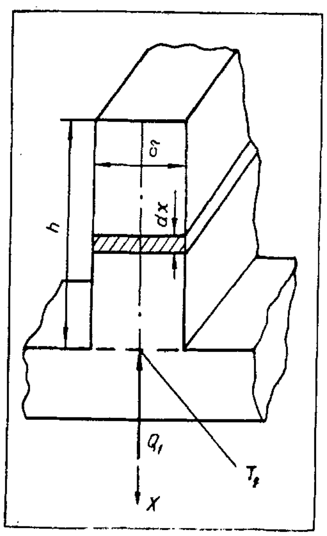

Fig. 1. Diagram of straight fin.

This paper presents a grapho-analytical method, of solving the problem of heat transfer through a surface equipped with longitudinal rectangular fins, allowing for variable thermal conductivity of the material.

A model of the physical system is shown in Fig. 1. The temperature distribution inside the fin is considered only as a function of distance from its base (this assumption is valid for relatively thin fins [1]). Then the temperature distribution along the fin is described by the following differential equation:

$$\frac{d}{dx}\left[\lambda(T)F\frac{dT}{dx}\right]-\alpha\Pi(T-T_0)=0, \tag{1}$$

or

$$\frac{d^2T}{dx^2}+\frac{d[\lambda(T)]}{\lambda(T)dT}\left(\frac{dT}{dx}\right)^2=\frac{\alpha\Pi}{F\lambda(T)}(T-T_0). \tag{1'}$$

We set

$$\alpha(T-T_0)=\frac{d}{dx}\left[\varphi(T)\right]; \tag{2}$$

then (1) takes the form

$$\frac{d}{dx}\left[\lambda(T)\frac{F}{\Pi}\frac{dT}{dx}\right]-\frac{d}{dx}[\varphi(T)]=0 \tag{3}$$

and

$$\frac{dT}{dx}=\frac{\Pi}{F}\frac{\varphi(T)+C_1}{\lambda(T)}. \tag{4}$$

Solving (4) together with (2) and integrating,

$$\int_{T_2}^{T}\varphi(T)\,d[\varphi(T)]+C_1\int_{T_2}^{T}d[\varphi(T)]=\alpha\frac{F}{\Pi}\int_{T_2}^{T}(T-T_0)\lambda(T)\,dT,$$

we obtain a reduced quadratic equation in $\varphi(T)$:

$$\varphi^2(T)+2C_1\varphi(T)+C_2=2\alpha\frac{F}{\Pi}\int_{T_2}^{T}(T-T_0)\lambda(T)\,dT, \tag{5}$$

where

$$C_2=-[2C_1\varphi(T_2)+\varphi^2(T_2)].$$

Whence

$$\varphi(T)=-C_1\pm\sqrt{C_1-C_2+2\alpha\frac{F}{\Pi}\int_{T_2}^{T}(T-T_0)\lambda(T)\,dT}.$$

Taking into account that for the chosen coordinate origin the temperature gradient dT/dx along the x axis increases, after substituting the expression for φ(T) in (4), we obtain

$$\frac{dT}{dx}=\frac{\Pi}{F\lambda(T)}\sqrt{C+2\alpha\frac{F}{\Pi}\int_{T_2}^{T}(T-T_0)\lambda(T)\,dT}, \tag{6}$$

where $C = C_1 - C_2$.

The arbitrary constant C can be determined from the boundary conditions

$$T = T_1 \text{ when } x = h,$$
$$T = T_2 \text{ when } x = 0, \ \frac{dT}{dx} = 0. \quad (7)$$

The second condition means that the end face of the fin is impermeable to heat. The end face has a small surface in comparison with the side of the fin, and therefore heat transfer from the end may be neglected. When heat transfer from the end of the fin can not be neglected, it is recommended in [2] that the fin height be increased by half its thickness.

Thus, from the second condition of (7) and (6) we find

$$C = -2\alpha \frac{F}{\Pi} \int_{T_2}^{T_2} (T - T_0) \lambda(T) dT = 0, \quad (8)$$

and from (6) we obtain the temperature gradient

$$\frac{dT}{dx} = \frac{\Pi}{F\lambda(T)} \sqrt{2\alpha \frac{F}{\Pi} \int_{T_2}^{T} (T - T_0) \lambda(T) dT}. \quad (9)$$

Returning to the basic parameters

$$\Pi = 2L + 2\delta \cong 2L, \ F = L\delta,$$

we find the heat flux at an arbitrary fin section

$$Q_f = \sqrt{4\alpha L^2 \delta \int_{T_2}^{T} (T - T_0) \lambda(T) dT}. \quad (10)$$

Under steady-state conditions, the amount of heat removed from the fin in unit time is equal to the amount of heat passing through the base of the fin, where x = h.

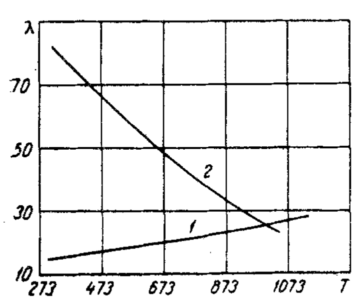

Fig. 2. Temperature dependence of thermal conductivity: (1) for EYalT; (2) for St. 20.

Then

$$Q_1 = \lambda(T) F_1 \left. \frac{dT}{dx} \right|_{x=h}, \quad (11)$$

or

$$Q_1 = \sqrt{4\alpha L^2 \delta \int_{T_2}^{T_1} (T - T_0) \lambda(T) dT}. \quad (12)$$

For relatively tall fins the problem of calculating the heat transfer through the fin is considerably simplified, since it may then be considered that the temperature at the end of the fin is equal to that of the surrounding medium, i.e., $T_2 = T_0$.

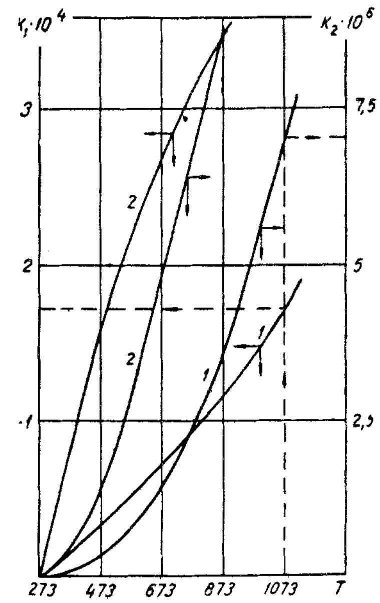

Fig. 3. Curves of K_1 and K_2 versus temperature: (1) for EYalT; (2) for St. 20.

After some transformations we obtain (12) in the form

$$Q_1 = \sqrt{4\alpha L^2 \delta \left\{ \int_{T_N}^{T_1} \lambda(T) T dT - T_0 \int_{T_N}^{T_1} \lambda(T) dT \right\}}, \quad (13)$$

where $T_N < T_x$, T_1.

Existing relations between thermal conductivity and temperature (Fig. 2) may be used to calculate the integrals $K_1 = \int_{T_N}^{T_x} \lambda(T) dT$ and $K_2 = \int_{T_N}^{T_x} \lambda(T) T dT$, written in the form of the approximating sums

$$K_1 = \sum_{i=1}^{n} \lambda(T_i) \Delta T_i, \quad K_2 = \sum_{i=1}^{n} \lambda(T_i) \Delta T_i T_i. \quad (14)$$

Curves showing K_1 and K_2 as functions of temperature for EYalT and St. 20 steel are given in Fig. 3. In constructing the curves it was assumed that $T_N = 0$ and $\Delta T_i = 20°K$, while $\lambda(T_i)$ was determined with the aid of Fig. 2.

The example presented below illustrates that it is possible to solve the heat transfer problem, with allowance for variation of the thermal conductivity of the material with temperature.

Example. To determine the amount of heat transferred through a fin ($l = 1$ m, $\delta = 0.01$ m, $h = 0.05$ m) with $\alpha = 3.6 \cdot 10^4$ W/m$^2 \cdot$ deg; $T_0 = 373°$ K for EYalT, when $T_1 = 1073°$ K and $\lambda(T) = 27.5$ W/m $\cdot$ deg., and for St. 20, when $T_1 = 873°$ K and $\lambda(T) = 33$ W/m $\cdot$ deg.

To determine the value of the heat flux through the fin with $\lambda = f(T)$ we use Fig. 3:

$$K_1 = 1.65 \cdot 10^4 \quad \text{for EYalT;}$$
$$K_2 = 7.12 \cdot 10^6 \quad \text{for EYalT;}$$

$$K_1 = 3.48 \cdot 10^4 \quad \text{for St. 20;}$$
$$K_2 = 8.76 \cdot 10^6 \quad \text{for St. 20.}$$

Then from (13) and [2] we find

$$\text{when } \lambda = f(T) \; Q_1 = 88\,800 \text{ W (EYalT);}$$
$$Q_1 = 87\,200 \text{ W (St. 20);}$$
$$\text{when } \lambda = \text{const} \; Q_1 = 98\,200 \text{ W (EYalT);}$$
$$Q_1 = 66\,900 \text{ W (St. 20).}$$

It is clear from the example that Q_1 must be determined with allowance for variation of the thermal conductivity of the material with temperature.

The values of K_1 and K_2 calculated according to (14) are the more accurate, the greater n. To estimate the error in K_1 and K_2, a calculation was performed at various ΔT_i for material St. 20 and a fin base temperature $T_1 = 873°$ K. The error in the range $\Delta T_i = 20°-50°$ K was about 1%. For practical calculations it is recommended that ΔT_i be in the range 20–40 degrees.

NOTATION

T—temperature; T_0—temperature of coolant; T_1—temperature at base of fin; T_N—some characteristic temperature; $\lambda(T)$—thermal conductivity of fin material; α—heat transfer coefficient; F—cross-sectional area of fin; Π—fin perimeter; h—fin height; L—fin length; δ—fin thickness; Q—heat flux; ΔT_i—change of temperature in i-th section; T_i—mean temperature at i-th section.

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